

Year 1

Addition

+ = signs and missing numbers

$$3 + 4 = \square \qquad \square = 3 + 4$$

$$3 + \square = 7 \qquad 7 = \square + 4$$

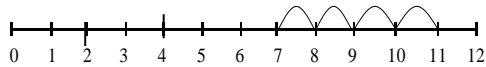
$$\square + 4 = 7 \qquad 7 = 3 + \square$$

$$\square + \nabla = 7 \qquad 7 = \square + \nabla$$

Promoting covering up of operations and numbers.

Number lines (numbered)

$$7 + 4$$



Recording by - drawing jumps on prepared lines

- constructing own lines

(Teacher model number lines with missing numbers)

(Teachers model jottings appropriate for larger numbers)

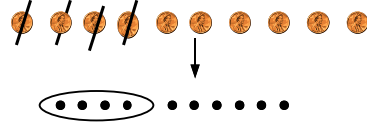
Mental Strategies

e.g. Know addition facts to 10

Subtraction

Pictures / marks

Sam spent 4p. What was his change from 10p?



- = signs and missing numbers

$$7 - 3 = \square \qquad \square = 7 - 3$$

$$7 - \square = 4 \qquad 4 = \square - 3$$

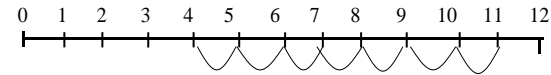
$$\square - 3 = 4 \qquad 4 = 7 - \square$$

$$\square - \nabla = 4 \qquad 4 = \square - \nabla$$

Number lines (numbered)

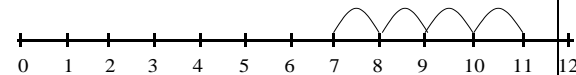
$$11 - 7$$

(Counting back)



The difference between 7 and 11

(Counting up)



Recording by - drawing jumps on prepared lines
- constructing own lines

(Teachers model jottings appropriate for larger numbers)

Mental Strategies

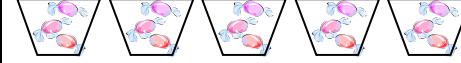
e.g. Know subtraction facts to 10

Multiplication

Pictures and symbols

There are 3 sweets in one bag.

How many sweets are there in 5 bags?



(Recording on a number line modelled by the teacher when solving problems)

Multiplication tables

Know facts for the 2 x tables

Division

Pictures and symbols

12 children get into teams of 4 to play a game. How many teams are there?



Year 2

Addition

+ = signs and missing numbers

Continue using a range of equations as in Year 1 but with appropriate, larger numbers.

Extend to

$$14 + 5 = 10 + \square$$

and adding three numbers

$$32 + \square + \square = 100 \quad 35 = 1 + \square + 5$$

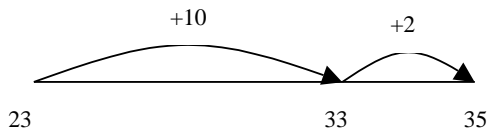
Partition into tens and ones and recombine

$$\begin{aligned} 12 + 23 &= 10 + 2 + 20 + 3 \\ &= 30 + 5 \\ &= 35 \end{aligned}$$

refine to partitioning the second number only:

$$\begin{aligned} 23 + 12 &= 23 + 10 + 2 \\ &= 33 + 2 \\ &= 35 \end{aligned}$$

Number lines (numbered then empty)

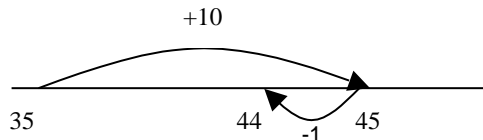


Mental Strategies

e.g. Know by heart addition facts to 10 (then 20)

Add 9 or 11 by adding 10 and adjusting by 1. Begin to add 19 or 21

$$35 + 9 = 44$$



Subtraction

- = signs and missing numbers

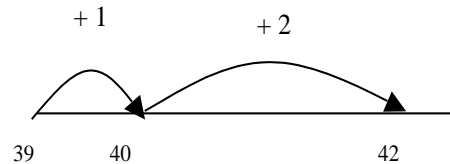
Continue using a range of equations as in Year 1 but with appropriate numbers.

Extend to $14 + 5 = 20 - \square$

Find a small difference by counting up

$$42 - 39 = 3$$

Number lines (numbered then empty)

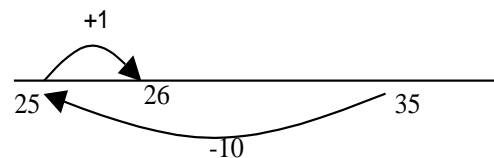


Mental Strategies

e.g. Know by heart subtraction facts to 10 (then 20)

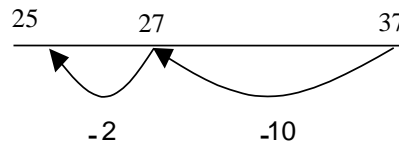
Subtract 9 or 11 by subtracting 10 and adjusting by 1. Begin to subtract 19 or 21

$$35 - 9 = 26$$



Use known number facts and place value to subtract (partition second number only)

$$\begin{aligned} 37 - 12 &= 37 - 10 - 2 \\ &= 27 - 2 \\ &= 25 \end{aligned}$$



Multiplication

Pictures and Symbols

There are 4 apples in one box. How many apples in 6 boxes?



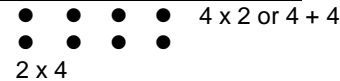
Multiplication tables

Know by heart facts for 2, 5 and 10 x tables

x = signs and missing numbers

$$\begin{aligned} 7 \times 2 &= \square & \square &= 2 \times 7 \\ 7 \times \square &= 14 & 14 &= \square \times 7 \\ \square \times 2 &= 14 & 14 &= 2 \times \square \\ \square \times \nabla &= 14 & 14 &= \square \times \nabla \end{aligned}$$

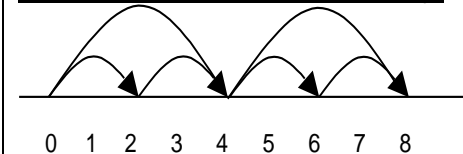
Arrays and repeated addition



or repeated addition

$$2 + 2 + 2 + 2$$

Number lines (numbered then empty)



Doubling by partitioning

$$15 \times 2$$

$$\begin{array}{r} 10 + 5 \\ \downarrow \quad \downarrow \times 2 \\ 20 + 10 = 30 \end{array}$$

OR

$$15 \times 2 = 30$$

$$10 \times 2 = 20$$

$$5 \times 2 = 10$$

$$20 + 10 = 30$$

Division

Pictures and Symbols

4 eggs fit in a box. How many boxes would you need to pack 20 eggs?



÷ = signs and missing numbers

$$\begin{aligned} 6 \div 2 &= \square & \square &= 6 \div 2 \\ 6 \div \square &= 3 & 3 &= 6 \div \square \\ \square \div 2 &= 3 & 3 &= \square \div 2 \\ \square \div \nabla &= 3 & 3 &= \square \div \nabla \end{aligned}$$

Understand division as sharing and grouping

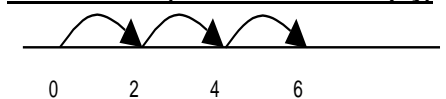
6 ÷ 2 can be modelled as:

Sharing – 6 sweets are shared between 2 people. How many do they have each?



Grouping – There are 6 sweets. How many people can have 2 each? (How many 2's make 6?)

Number lines (numbered then empty)



Halving by partitioning

$$16 \div 2$$

$$\begin{array}{r} 10 + 6 \\ \downarrow \quad \downarrow \div 2 \\ 5 + 3 = 8 \end{array}$$

OR $16 \div 2 = 8$

$$10 \div 2 = 5$$

$$6 \div 2 = 3$$

$$5 + 3 = 8$$

Year 3

Addition

+ = signs and missing numbers

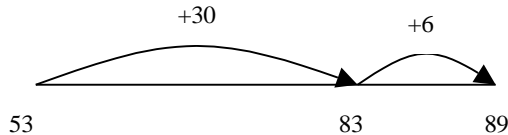
Continue using a range of equations as in Year 1 and 2 but with appropriate, larger numbers.

Partition into tens and ones and recombine

Partition both numbers and recombine. Refine to partitioning the second number only e.g.

$$\begin{aligned} 36 + 53 &= 53 + 30 + 6 \\ &= 83 + 6 \\ &= 89 \end{aligned}$$

Number Lines



Mental Strategies

e.g. Know by heart addition facts to 20

Add a near multiple of 10 to a two-digit number (Continue as in Year 2 but with appropriate numbers e.g. $35 + 19$ is the same as $35 + 20 - 1$).

Pencil and paper procedures

(TU + TU then HTU + TU, HTU + HTU)

$$83 + 42 = 125$$

leading to:	
83	83
+ 42	+ 42
120	5
5	120
125	125

Subtraction

- = signs and missing numbers

Continue using a range of equations as in Year 1 and 2 but with appropriate numbers.

Find a small difference by counting up

Continue as in Year 2 but with appropriate numbers e.g. $102 - 97 = 5$

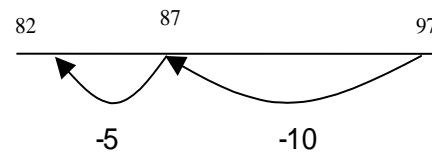
Mental Strategies

e.g. Know by heart subtraction facts to 20.

Subtract mentally a 'near multiple of 10' to or from a two-digit number (Continue as in Year 2 but with appropriate numbers e.g. $78 - 49$ is the same as $78 - 50 + 1$).

Use known number facts and place value to subtract (Continue as in Year 2 but with appropriate numbers e.g. $97 - 15 = 82$).

Number Lines

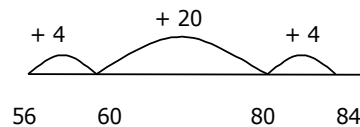


Pencil and paper procedures

(TU-TU then HTU-TU, HTU-HTU)

Complementary addition

$$84 - 56 = 28$$



Multiplication

Pictures and Markings

A spider has 8 legs.

How many legs do 4 spiders have?



Multiplication tables

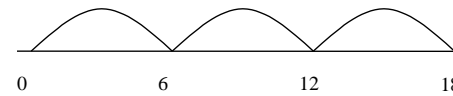
Know by heart facts for the 2, 3, 4, 5, 6, 10 x tables

x = signs and missing numbers

Continue using a range of equations as in Year 2 but with appropriate numbers.

Number lines

$$6 \times 3$$



Arrays and repeated addition

Continue to understand multiplication as repeated addition and continue to use arrays (as in Year 2).

Doubling by partitioning

$$\begin{aligned} 35 \times 2 &= 70 \\ 30 \times 2 &= 60 \\ 5 \times 2 &= 10 \\ 60 + 10 &= 70 \end{aligned}$$

OR

$$\begin{array}{r} 30 \\ + 5 \\ \hline 35 \end{array} \times 2$$

$$60 + 10 = 70$$

Use known facts and place value to carry out simple multiplications

e.g. $32 \times 3 = 96$

$$\begin{array}{r} 30 \\ + 2 \\ \hline 32 \end{array} \times 3$$

$$90 + 6 = 96$$

Division

Pictures and Markings

8 children can travel in a minibus.

How many minibuses would you need to take 29 children to a football match?



÷ = signs and missing numbers

Continue using a range of equations as in Year 2 but with appropriate numbers.

Understand division as sharing and grouping

Divisions can be modelled as:

Sharing – 15 shared between 3 (see Year 2 diagram)

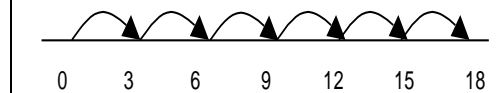
Grouping - How many 3s make 18?

Number Lines

Sharing



Grouping



Halving by partitioning

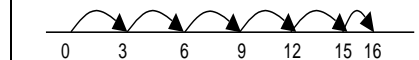
As Year 2 but with appropriate numbers

Remainders

$$16 \div 3 = 5 \text{ r}1$$

Sharing - 16 shared between 3, how many left over?

Grouping – How many 3s make 16, how many left over? e.g.



Year 4

Addition

+ = signs and missing numbers

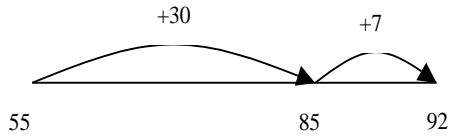
Continue using a range of equations as in Year 1 and 2 but with appropriate numbers.

Partition into tens and ones and recombine

Either partition both numbers and recombine or partition the second number only e.g.

$$\begin{aligned} 55 + 37 &= 55 + 30 + 7 \\ &= 85 + 7 \\ &= 92 \end{aligned}$$

Number lines



Mental Strategies

e.g. Add the nearest multiple of 10, then adjust (Continue as in Year 2 and 3 but with appropriate numbers e.g. $63 + 29$ is the same as $63 + 30 - 1$)

Pencil and paper procedures

(HTU+TU then HTU+HTU)

e.g. $358 + 73$

$$\begin{array}{r} 358 \\ + 73 \\ \hline 11 \\ 120 \\ \hline 300 \\ \hline 431 \end{array}$$

leading to

$$\begin{array}{r} 358 \\ + 73 \\ \hline 431 \\ \hline \end{array}$$

Extend to decimals in the context of money (vertically)
(Revert to expanded methods if the children experience any difficulty)

Subtraction

- = signs and missing numbers

Continue using a range of equations as in Year 1 and 2 but with appropriate numbers.

Find a small difference by counting up

e.g. $5003 - 4996 = 7$

This can be modelled on an empty number line (see complementary addition below).

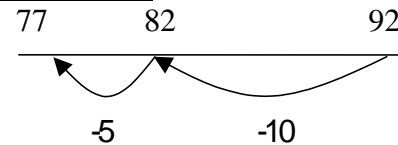
Mental Strategies

e.g. Subtract the nearest multiple of 10, then adjust (Continue as in Year 2 and 3 but with appropriate numbers).

Use known number facts and place value to subtract_ e.g. $92 - 15 = 77$

$$\begin{aligned} 92 - 10 &= 82 \\ 82 - 5 &= 77 \end{aligned}$$

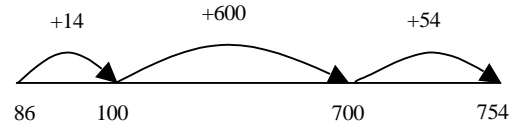
Number lines



Pencil and paper procedures

(HTU-TU then HTU-HTU)

Complementary addition e.g. $754 - 86 = 668$



754	OR	754
- 86		- 86
4 → 90		14 → 100
10 → 100		600 → 700
600 → 700		54 → 754
50 → 750		54 → 754
4 → 754		668
668		

(Extend to decimals in the context of money)
Extend to decomposition for HA

Multiplication

x = signs and missing numbers

Continue using a range of equations as in Year 2 but with appropriate numbers

Multiplication tables

Know by heart facts for x tables (up to 10x)

Partitioning

$23 \times 4 = 92$

$$\begin{aligned} 23 \times 4 &= (20 \times 4) + (3 \times 4) \\ &= (80) + (12) \\ &= 92 \end{aligned}$$

Pencil and paper procedures

(TU x U)

Grid method

23×7 is approximately $20 \times 10 = 200$

x	20	3	=	161
7	140	21		

x	70	2	=	2160
30	2100	60		
8	560	16	+	576
				2736

Developing standard written method

23	leading to	23
x 4		x 4
80 (4 x 20)		12 (4 x 3)
12 (4 x 3)		80 (4 x 20)
92		92
leading to		
23		
x 4		
92		
2736		

Division

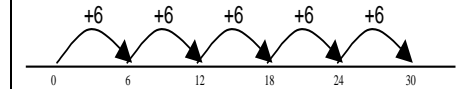
÷ = signs and missing numbers

Continue using a range of equations as in Year 2 but with appropriate numbers.

Sharing and grouping

$30 \div 6$ can be modelled as:

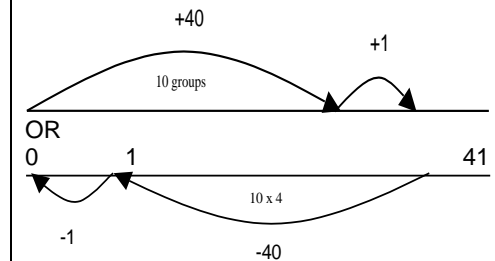
Grouping – groups of 6 taken away and the number of groups counted e.g.



Sharing – sharing among 6, the number given to each person

Remainders

$41 \div 4 = 10 \text{ r}1$



OR $41 = (10 \times 4) + 1$

Pencil and paper procedures (TU ÷ U)

$72 \div 5$ lies between $50 \div 5 = 10$ and $100 \div 5 = 20$

Chunking

$$\begin{array}{r} 72 \\ - 50 \quad (10 \text{ groups}) \text{ or } (10 \times 5) \\ \hline 22 \\ - 20 \quad (4 \text{ groups}) \text{ or } (4 \times 5) \\ \hline 2 \end{array}$$

Answer : 14 remainder 2

Year 5

Addition

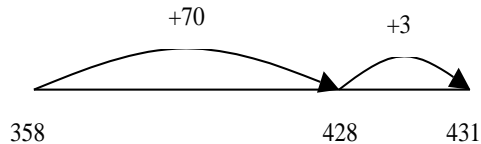
+ = signs and missing numbers

Continue using a range of equations as in Year 1 and 2 but with appropriate numbers.

Partition into hundreds, tens and ones and recombine

Either partition both numbers and recombine or partition the second number only e.g.

$$\begin{aligned} 358 + 73 &= 358 + 70 + 3 \\ &= 428 + 3 \\ &= 431 \end{aligned}$$



Mental Strategies

e.g. Add the nearest multiple of 10 or 100, then adjust (Continue as in Year 2, 3 and 4 but with appropriate numbers e.g. $458 + 79 =$ is the same as $458 + 80 - 1$).

Pencil and paper procedures

(HTU + HTU then ThHTU + ThHTU)

Formal method, showing numbers carried underneath

$$\begin{array}{r} 358 \\ + 73 \\ \hline 431 \\ 11 \end{array}$$

Extend to numbers with at least four digits
 $3587 + 675 = 4262$

$$\begin{array}{r} 3587 \\ + 675 \\ \hline 4262 \\ 111 \end{array}$$

Revert to expanded methods if the children experience any difficulty.

Extend to decimals (same number of decimal places) and adding several numbers (with different numbers of digits).

Model negative numbers using a number line.

Subtraction

- = signs and missing numbers

Continue using a range of equations as in Year 1 and 2 but with appropriate numbers.

Find a difference by counting up

e.g. $8006 - 2993 = 5013$

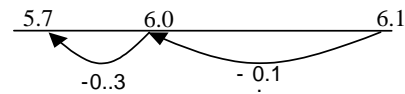
This can be modelled on an empty number line (see complementary addition below).

Mental Strategies

e.g. Subtract the nearest multiple of 10 or 100, then adjust (Continue as in Year 2, 3 and 4 but with appropriate numbers).

Use known number facts and place value to subtract

$$6.1 - 0.4 = 5.7$$



Pencil and paper procedures

(HTU - HTU then ThHTU - ThHTU)

Complementary addition

$$754 - 286 = 468$$

Using an empty number line (see Year 4)

OR

$$\begin{array}{r} 754 \\ - 286 \\ \hline 14 \rightarrow 300 \text{ can be refined to} \\ 400 \rightarrow 700 \\ \underline{54} \rightarrow 754 \\ \hline 468 \end{array} \qquad \begin{array}{r} 754 \\ - 286 \\ \hline 14 \rightarrow 300 \\ \underline{454} \rightarrow 754 \\ \hline 468 \end{array}$$

Decomposition (teacher uses expanded form first to explain)

$$\begin{aligned} &600 + 140 \\ &\quad 40 + 14 \\ 754 &= 700 + 50 + 4 \\ -286 &\quad 200 + 80 + 6 \\ \hline &400 + 60 + 8 \end{aligned} \qquad \begin{array}{r} 6714514 \\ - 286 \\ \hline 468 \end{array}$$

Extend to decimals (same number of decimal places). Check sums using inverse operation.

Multiplication

x = signs and missing numbers

Continue using a range of equations as in Year 2 but with appropriate numbers

Partitioning

$$47 \times 6 = 92$$

$$\begin{aligned} 47 \times 6 &= (40 \times 6) + (7 \times 6) \\ &= (240) + (42) \\ &= 282 \end{aligned}$$

OR

Use the grid method of multiplication (as below)

Pencil and paper procedures

(HTU x U, TU x TU)

Grid method

72×38 is approximately $70 \times 40 = 2800$

x	70	2	
30	2100	60	2160
8	560	16	+ 576
			<u>2736</u>
			1

Standard Written Method

$$\begin{array}{r} 346 \\ \times 9 \\ \hline 54 \quad (9 \times 6) \text{ leading to } 3114 \\ 360 \quad (9 \times 40) \\ \underline{2700} \quad (9 \times 300) \\ 3114 \end{array} \qquad \begin{array}{r} 346 \\ \times 9 \\ \hline 3114 \\ 45 \end{array}$$

$$\begin{array}{r} 87 \quad \text{leading to} \quad 87 \\ \times 26 \\ \hline 140 \quad (20 \times 7) \\ 1600 \quad (20 \times 80) \\ 42 \quad (6 \times 7) \\ \underline{480} \quad (6 \times 80) \\ 2262 \end{array} \qquad \begin{array}{r} 87 \\ \times 26 \\ \hline 1740 \quad (20 \times 87) \\ 11 \\ + 522 \quad (6 \times 87) \\ \hline 2262 \\ 1 \end{array}$$

Extend to multiplying decimals fractions with one decimal place by a single digit.

Division

÷ = signs and missing numbers

Continue using a range of equations as in Year 2 but with appropriate numbers.

Sharing and grouping

Continue to understand division as both sharing and grouping (repeated subtraction).

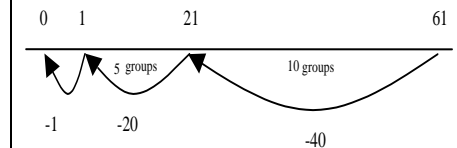
Remainders

Quotients expressed as fractions or decimal fractions

$$61 \div 4 = 15 \frac{1}{4} \text{ or } 15.25$$



OR



Pencil and paper procedures (HTU ÷ U)

$256 \div 7$ lies between $210 \div 7 = 30$ and $280 \div 7 = 40$

Standard Written Method

$$\begin{array}{r} 36 \text{ r } 4 \\ 7 \overline{)256} \\ \underline{-70} \quad (10 \text{ groups}) \text{ or } (10 \times 7) \\ 186 \\ \underline{-140} \quad (20 \text{ groups}) \text{ or } (20 \times 7) \\ 46 \\ \underline{-42} \quad (6 \text{ groups}) \text{ or } (6 \times 7) \\ 4 \end{array}$$

Compact short division for HA.

Year 6

Addition

+ = signs and missing numbers

Continue using a range of equations as in Year 1 and 2 but with appropriate numbers.

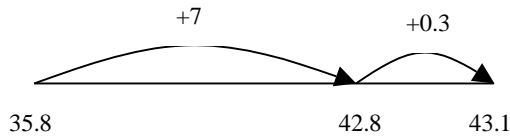
Partition into hundreds, tens, ones and decimal fractions and recombine

Either partition both numbers and recombine or partition the second number only e.g.

$$35.8 + 7.3 = 35.8 + 7 + 0.3$$

$$= 42.8 + 0.3$$

$$= 43.1$$



Mental Strategies

e.g. Add the nearest multiple of 10, 100 or 1000, then adjust (Continue as in Year 2, 3, 4 and 5 but with appropriate numbers including extending to adding 0.9, 1.9, 2.9 etc.)

Pencil and paper procedures

(ThHTU + ThHTU)

Extend to numbers with any number of digits and decimals with 1 and 2 decimal places.
 $124.9 + 117.25 = 242.15$

$$\begin{array}{r} 124.9 \\ + 117.25 \\ \hline 242.15 \\ \text{11} \end{array}$$

Revert to expanded methods if the children experience any difficulty.
 Extend to decimals (either one or two decimal places).

Subtraction

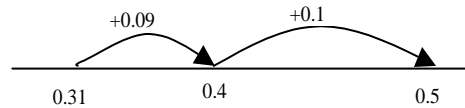
- = signs and missing numbers

Continue using a range of equations as in Year 1 and 2 but with appropriate numbers.

Find a difference by counting up

e.g. $0.5 - 0.31 = 0.19$

This can be modelled on an empty number line (see complementary addition below).



Mental Strategies

e.g. Subtract the nearest multiple of 10, 100 or 1000, then adjust (Continue as in Year 2, 3, 4 and 5 but with appropriate numbers).

Use known number facts and place value to subtract (Continue as Year 5)

Pencil and paper procedures

(ThHTU - ThHTU then any number of digits)

Complementary addition (used if children experience difficulty with decomposition)

$$6467 - 2684 = 3783$$

Using an empty number line (see Year 4)

OR

6467	refined to	6467
$- 2684$		$- 2684$
$16 \rightarrow 2700$		$316 \rightarrow 3000$
$300 \rightarrow 3000$		$316 \rightarrow 3000$
$3467 \rightarrow 6467$		$3467 \rightarrow 6467$
3783		3783

Decomposition

$$\begin{array}{r} 5 \text{ 13 1} \\ 6467 \\ - 2684 \\ \hline 3783 \end{array} \quad (\text{check calculations using inverse})$$

Revert to expanded methods if children experience any difficulty

Extend to decimal fractions (1 or 2 dec. places)

Multiplication

x = signs and missing numbers

Continue using a range of equations as in Year 2 but with appropriate numbers

Partitioning

$$87 \times 6 = 522$$

$$87 \times 6 = (80 \times 6) + (7 \times 6)$$

$$= (480) + (42)$$

$$= 522$$

OR

Use the grid method of multiplication (as below)

Pencil and paper procedures

(ThHTU x U, HTU x TU)

Grid method

372×24 is approximately $400 \times 20 = 8000$

x	300	70	2	
20	6000	1400	40	7440
4	1200	280	8	$+1488$
				$\underline{8928}$
				1

Standard Written Method

(approximate first)

4346	352	
$x \quad 8$	$x \quad 27$	
$\underline{34768}$	$\underline{7040}$	(20×352)
234	704	
	1	
	$+ 2464$	(7×352)
	31	
	$\underline{9504}$	
	1	

Extend to multiplying decimals with up to two decimal places by a single digit.

Division

÷ = signs and missing numbers

Continue using a range of equations as in Year 2 but with appropriate numbers.

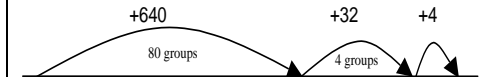
Sharing and grouping

Continue to understand division as both sharing and grouping (repeated subtraction).

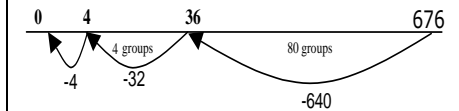
Remainders

Quotients expressed as fractions or decimal fractions

$$676 \div 8 = 84.5$$



OR



Pencil and paper procedures (HTU ÷ TU)

$977 \div 36$ is approximately $1000 \div 40 = 25$

Standard Written Method

$\begin{array}{r} 27 \text{ } ^5/_{36} \\ 36 \overline{) 977} \\ - 360 \quad (10 \times 36) \\ \hline 617 \\ - 617 \quad (10 \times 36) \\ \hline 257 \\ - 180 \quad (5 \times 36) \\ \hline 77 \\ - 72 \quad (2 \times 36) \\ \hline 5 \end{array}$	$\begin{array}{r} 27 \text{ } ^5/_{36} \\ 36 \overline{) 977} \\ - 720 \quad (20x) \\ \hline 257 \\ - 180 \quad (5x) \\ \hline 77 \\ - 72 \quad (2x) \\ \hline 5 \end{array}$
--	---

Extend to decimal fractions with up to 2 decimal places

Extend to efficient compact division for HA.

